

REFLECTION OF LIGHT FROM  
MULTI-LAYER FILMS

by  
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Feynman had developed an appetite for new problems—any problems. He would stop people he knew in the corridor of the physics building and ask what they were working on. They quickly discovered that the question was not the usual small talk. Feynman pushed for details. He caught one classmate, Monarch Cutler, in despair. Cutler had taken on a senior thesis problem based on an important discovery in 1938 by two professors in the optics laboratory. They found that they could transform the refracting and reflecting qualities of lenses by evaporating salts onto them, forming very thin coatings, just a few atoms thick. Such coatings became essential to reducing unwanted glare in the lenses of cameras and telescopes. Cutler was supposed to find a way of calculating what happened when different thin films were applied, one atop another. His professors wondered, for example, whether there was a way to make exceedingly pure color filters, passing only light of a certain wavelength. Cutler was stymied. Classical optics should have sufficed—no peculiarly quantum effects came into play—but no one had ever analyzed the behavior of light passing through a parade of mostly transparent films thinner than a single wavelength. Cutler told Feynman he could find no literature on the subject. He did not know where to start. A few days later Feynman returned with the solution: a formula summing an infinite series of reflections back and forth from the inner surfaces of the coatings. He showed how the combinations of refraction and reflection would affect the phase of the light, changing its color. Using Feynman's theory and many hours on the Marchant calculator, Cutler also found a way to make the color filters his professors wanted.

Developing a theory for reflection by multiple-layer thin films was not so different for Feynman from math team in the now-distant past of Far Rockaway. He could see, or feel, the intertwined infinities of the problem, the beam of light resonating back and forth between the pair of surfaces, and then the next pair, and so on, and he had a giant



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## PREFACE

This thesis is the result of work leading to the degree of Bachelor of Science in Physics.

Recently Dr. Hawley C. Cartwright and Dr. Arthur F. Turner proposed the placing of thin films of varying indices on glass to produce color filters through the interference of light by these films. This paper is a theoretical study of the reflection and transmission characteristics of these proposed filters. In Part I, equations are derived to enable the calculation of the reflection and transmission curves of multi-layer films. In Part II, these equations are applied to the special case of the color filters. In the Appendix are several tables and charts that were computed in the early stages of this work. They were not found to be as useful as expected and are included here only because it is believed that similar material is not available at present in other sources.

Grateful acknowledgment is extended to Professor Arthur C. Hardy, my thesis advisor, to Dr. Cartwright and Dr. Turner for information concerning their work in the production of the color filters, and to Richard P. Feynman for his aid in connection with Part I.

Monarch L. Cutler

Cambridge, Mass.,  
May, 1939.

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## PART I

### GENERAL SOLUTION FOR THE DETERMINATION OF REFLECTION FROM MULTI-LAYER FILMS

Reflection from a thin film is basically a problem in the interference of light. Essentially, the amplitude of the incident vibrating light is divided into two sections by a combination of reflection and refraction at the first surface of the film. These two sections are later reunited, this time by reflection from the lower surface, to produce interference.

#### THE VECTOR METHOD OF ANALYSIS

If a plane wave is incident on the surface 1-2 of the film in figure 1, the wave reflected at this surface will interfere with the one reflected at surface 34 in the plane ab (or a'b'). The difference in the phase of these two waves is  $2\theta$ .

$$2\theta = 4\pi nd \cos r/n_0\lambda_0; \quad (1)$$

where:  $d$  = thickness of film,

$r$  = angle of refraction,

$n_0$  = index of first medium,

$n$  = index of reflecting medium,

$\lambda_0$  = wavelength of the light in the first medium.



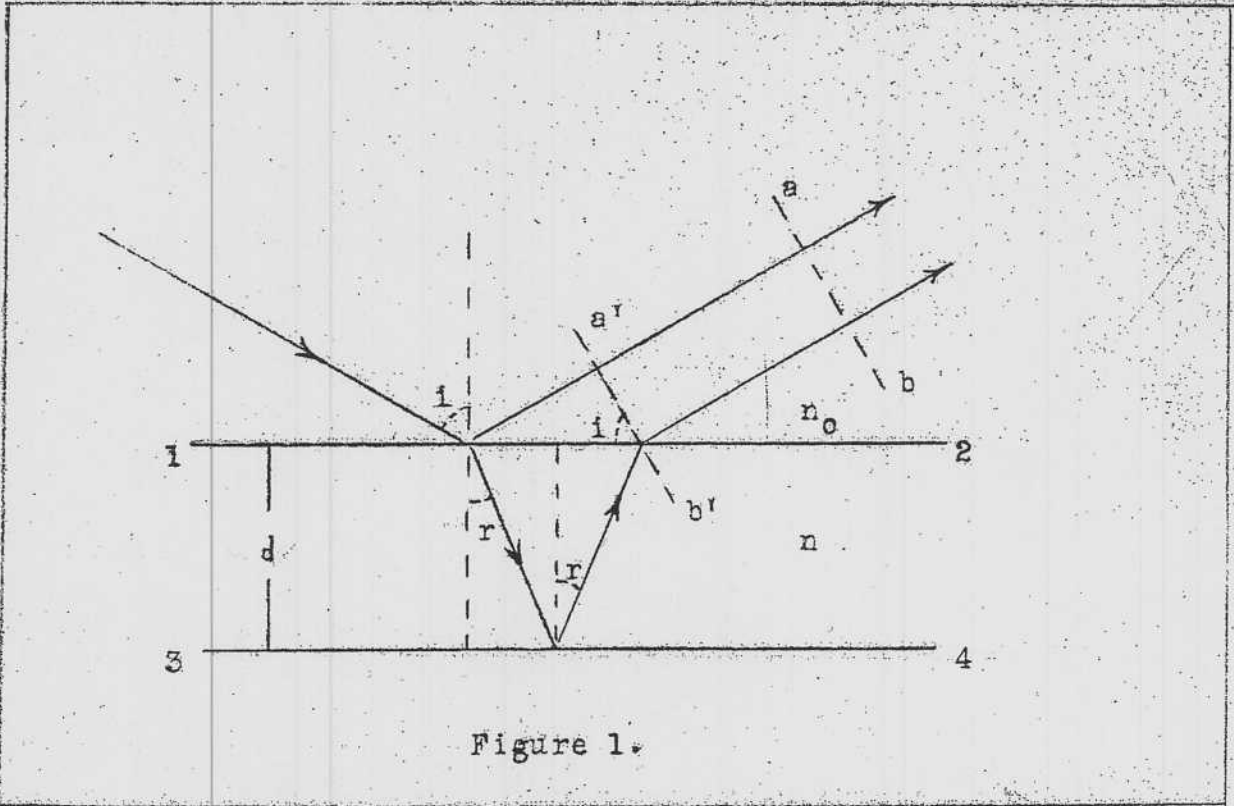


Figure 1.

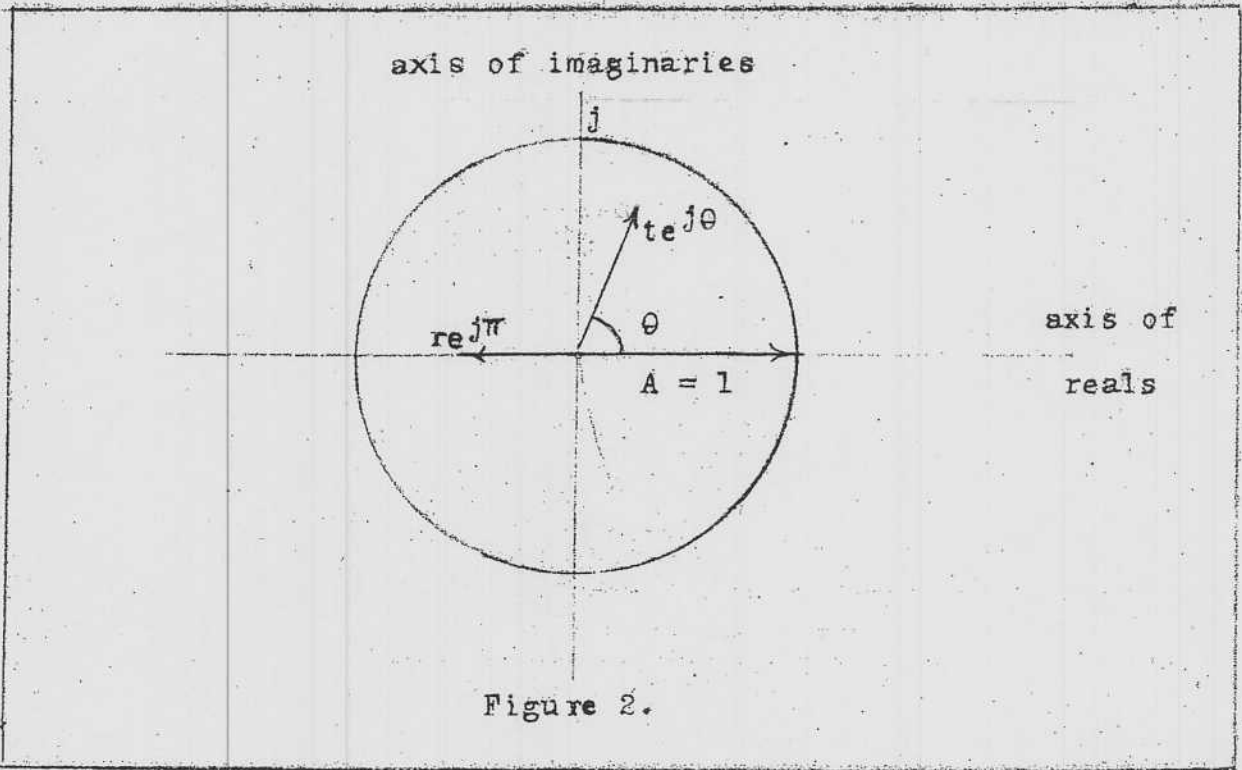


Figure 2.

